1.) Let $\boldsymbol{W} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ and $\boldsymbol{P} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find $\boldsymbol{WP}$.

(3)

(b) Given that $2\boldsymbol{WP} + \boldsymbol{S} = \begin{pmatrix} 26 \\ 12 \\ 10 \end{pmatrix}$, find $\boldsymbol{S}$.

(3)
(Total 6 marks)

2.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$.

(3)

(b) Solve $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B}$.

(2)
(Total 5 marks)

3.) Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$.

(a) (i) Find $\boldsymbol{AB}$.

(ii) Write down the inverse of $\boldsymbol{A}$.

(3)

Let $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

(b) Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$.

(4)
(Total 7 marks)

4.) A matrix $\boldsymbol{M}$ has **inverse** $\boldsymbol{M}^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$.

(a) Find $\boldsymbol{M}$.

(3)

(b) Solve the matrix equation $\boldsymbol{MX} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(3)
(Total 6 marks)

5.) Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -3 & 0 \\ 4 & -2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

(b) Let $\boldsymbol{B}$ be a 3 × 3 matrix. Given that $\boldsymbol{AB} + \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}$, find $\boldsymbol{B}$.

(4)
(Total 6 marks)

6.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

Find

(a) $\boldsymbol{A} + \boldsymbol{B}$;

(2)

(b) $-3\boldsymbol{A}$;

(2)

(c) $\boldsymbol{AB}$.

(3)
(Total 7 marks)

7.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$ in terms of p and q.

(2)

(b) Matrix $\boldsymbol{B}$ is the inverse of matrix $\boldsymbol{A}$. Find the value of p and of q.

(5)
(Total 7 marks)

8.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

The matrix $\boldsymbol{B}$ satisfies the equation $\left(\boldsymbol{I} - \frac{1}{2}\boldsymbol{B}\right)^{-1} = \boldsymbol{A}$, where $\boldsymbol{I}$ is the 3×3 identity matrix.

(b) (i) Show that $\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$.

(ii) Find $\boldsymbol{B}$.

(iii) Write down det $\boldsymbol{B}$.

(iv) **Hence**, explain why $\boldsymbol{B}^{-1}$ exists.

(6)

Let $\boldsymbol{BX} = \boldsymbol{C}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find $\boldsymbol{X}$.

(ii) Write down a system of equations whose solution is represented by $\boldsymbol{X}$.

(5)
(Total 13 marks)

9.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$.

(a) Find $\boldsymbol{A}^2$.

(2)

(b) Let $\boldsymbol{B} = \begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$. Solve the matrix equation $3\boldsymbol{X} + \boldsymbol{A} = \boldsymbol{B}$.

(3)
(Total 5 marks)

10.) Let $\boldsymbol{M} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$, and $\boldsymbol{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Given that $\boldsymbol{M}^2 - 6\boldsymbol{M} + k\boldsymbol{I} = \boldsymbol{O}$, find k.

(Total 6 marks)

11.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & x & -1 \\ 3 & 1 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 \\ x \\ 2 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$.

(b) The matrix $\boldsymbol{C} = \begin{pmatrix} 20 \\ 28 \end{pmatrix}$ and $2\boldsymbol{AB} = \boldsymbol{C}$. Find the value of x.

(Total 6 marks)

12.) Let $\boldsymbol{A} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) Find

(i) $\boldsymbol{A}^{-1}$;

(ii) $\boldsymbol{A}^2$.

(4)

Let $\boldsymbol{B} = \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix}$.

(b) Given that $2\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$, find the value of p and of q.

(3)

(c) Hence find $\boldsymbol{A}^{-1}\boldsymbol{B}$.

(2)

(d) Let $\boldsymbol{X}$ be a 2×2 matrix such that $\boldsymbol{AX} = \boldsymbol{B}$. Find $\boldsymbol{X}$.

(2)
(Total 11 marks)

13.)

(a) Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

(i) Write down the value of a.

(ii) Find the value of b.

(b) Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ 9 & 23 \end{pmatrix}$.

Find the value of q.

(Total 6 marks)

14.) Let S_n be the sum of the first n terms of the arithmetic series 2 + 4 + 6 + ….

(a) Find

(i) S_4;

(ii) S_{100}.

(4)

Let $\boldsymbol{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(b) (i) Find $\boldsymbol{M}^2$.

(ii) Show that $\boldsymbol{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$.

(5)

It may now be assumed that $\boldsymbol{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$, for $n \geq 4$. The sum $\boldsymbol{T}_n$ is defined by

$$\boldsymbol{T}_n = \boldsymbol{M}^1 + \boldsymbol{M}^2 + \boldsymbol{M}^3 + \ldots + \boldsymbol{M}^n .$$

(c) (i) Write down $\boldsymbol{M}^4$.

(ii) Find $\boldsymbol{T}_4$.

(4)

(d) Using your results from part (a) (ii), find $\boldsymbol{T}_{100}$.

(3)
(Total 16 marks)

15.) Let $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ k & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find, in terms of k,

(a) $2\boldsymbol{A} - \boldsymbol{B}$;

(b) $\det(2\boldsymbol{A} - \boldsymbol{B})$.

(Total 6 marks)

16.) Let $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$. Giving your answers in terms of a, b, c, d and e,

(a) write down $\boldsymbol{A} + \boldsymbol{B}$;

(b) find $\boldsymbol{AB}$.

(Total 6 marks)

17.) Let $C = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}$ and $\boldsymbol{D} = \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$.

The 2×2 matrix $\boldsymbol{Q}$ is such that $3\boldsymbol{Q} = 2\boldsymbol{C} - \boldsymbol{D}$

(a) Find $\boldsymbol{Q}$.

(3)

(b) Find $\boldsymbol{CD}$.

(4)

(c) Find $\boldsymbol{D}^{-1}$.

(2)
(Total 9 marks)

18.) Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are defined by

$\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 7 & 2 \end{pmatrix}$ $\boldsymbol{B} = \begin{pmatrix} 2 & 4 \\ -3 & 15 \end{pmatrix}$ $\boldsymbol{C} = \begin{pmatrix} 9 & -7 \\ 8 & 2 \end{pmatrix}$.

Let $\boldsymbol{X}$ be an unknown 2×2 matrix satisfying the equation

$$\boldsymbol{AX} + \boldsymbol{B} = \boldsymbol{C}.$$

This equation may be solved for $\boldsymbol{X}$ by rewriting it in the form

$$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{D}.$$

where $\boldsymbol{D}$ is a 2×2 matrix.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

(b) Find $\boldsymbol{D}$.

(3)

(c) Find $\boldsymbol{X}$.

(2)
(Total 7 marks)

19.) The matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{X}$ are given by

$\boldsymbol{A} = \begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$, $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \mathbb{Q}$.

Given that $\boldsymbol{AX} + \boldsymbol{X} = \boldsymbol{B}$, find the **exact** values of a, b, c and d.

(Total 8 marks)

20.) Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix}$.

(a) Write down the inverse, $\boldsymbol{A}^{-1}$.

(2)

(b) $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ are also 2×2 matrices.

(i) Given that $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C}$, express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$, $\boldsymbol{B}$ and $\boldsymbol{C}$.

(ii) Given that $\boldsymbol{B} = \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix}$, and $\boldsymbol{C} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$, find $\boldsymbol{X}$.

(4)
(Total 6 marks)

21.) Let $M = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$, where a $\in \mathbb{Z}$.

(a) Find M^2 in terms of a.

(4)

(b) If M^2 is equal to $\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$, find the value of a.

(2)

(c) Using this value of a, find M^{-1} and **hence** solve the system of equations:

$$-x + 2y = -3$$
$$2x - y = 3$$

(6)
(Total 12 marks)